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AUTHOR O'Brien, Peter; Richardson, Arthur
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ABSTRACT

This report: (1) describes the mathematics test given to Year 8 students in the Mt. Druitt Longitudinal Study in 1986; (2) describes and discusses the results; (3) analyzes errors made by the students; and (4) suggests some possible interventions to address the errors made by students. The study dates from 1975 when funds were obtained to provide intervention programs at five preschools in Mt. Druitt (Australia), an area which had been identified as being disadvantaged. These intervention programs were aimed at bridging the educational achievement gap between these "disadvantaged" students and those who were not "disadvantaged." The intervention programs ceased in 1979 when the children had completed first grade. Since that time the study has been involved in assessing educational skills, pupil attitudes, and teacher and pupil classroom behavior through testing, questionnaires, and observation. Teacher and parent interviews have also been conducted. To determine whether the early intervention assisted the Mt. Druitt students and how that "disadvantage" operated, comparison schools were chosen to undergo the same testing as the Mt. Druitt schools. In 1986, the students who were first identified in preschool in 1977 were in year eight in high school. Scripts of the 1986 test, which was administered in two parts on two separate occasions during a school week in March, are appended. (PK)

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MT. DRUITT LONGITUDINAL STUDY

1986 MATHEMATICS INVESTIGATION

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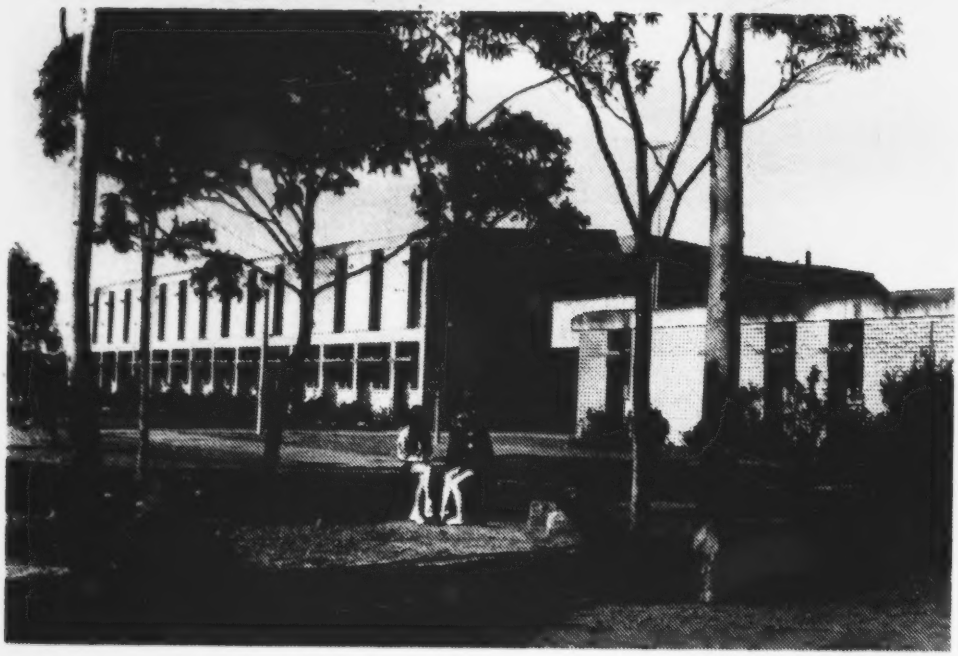

Peter O'Brien and Arthur Richardson

SCHOOL OF EDUCATION ● MACQUARIE UNIVERSITY, AUSTRALIA 2109

June, 1987

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MT DRUITT LONGITUDINAL STUDY

1986 MATHEMATICS INVESTIGATION

A Report on results,
including an analysis of errors and
suggestions for remediation

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BACKGROUND TO THE MT. DRUITT LONGITUDINAL STUDY

The Study dates from 1975 when funds were obtained from the Bernard van Leer Foundation to provide intervention programs at five preschools in Mt. Druitt, west of Sydney, an area which had been identified, by welfare agencies, as being disadvantaged. All Mt. Druitt schools were at that time identified as disadvantaged. These intervention programs were aimed at bridging the educational achievement gap between these "disadvantaged" students and those who were not "disadvantaged". The intervention programs ceased in 1979 when the children had completed first grade. Since that time the Study has been involved in assessing educational skills, pupil attitudes and teacher and pupil classroom behaviour through testing, questionnaires and observation. As well teacher and parent interviews have been conducted.

To determine whether the early intervention assisted the Mt. Druitt students and how it was that "disadvantage" operated, comparison schools were chosen to undergo the same testing as the Mt. Druitt schools. At present, 1986, the students who were first identified in preschool in 1977, are in Year 8 at High School and the Study has students in three high schools in the Mt. Druitt area, and in two non-disadvantaged contrast high schools, one of which is near Sydney whilst the other is distant from Sydney. The number of students from these two contrast high schools who were in the study in primary school has reduced greatly, due largely to many of them going to private schools. There are significant problems associated with longitudinal studies, one is the loss of students from the study through reasons such as changing schools and moving to a different area. To counter such losses, extra students were included in the sample in 1986 and a new, high school based longitudinal study commenced. It is planned that this study should continue for at least two years.

TECHNICAL REPORT ON 1986 MATHS INVESTIGATION

The aim of this report is : to describe the Maths test given to the students in the Mt. Druitt Longitudinal Study in 1986,
: to describe and discuss the results,
: to analyse the errors made by the students,
: to suggest some possible interventions to address the errors made by the students.

The Purpose of the Test

The test aimed at providing a means of comparing the achievement of the students both within and across the Year 8 sample as well as providing a basis for comparison of maths achievement over a period. In addition the test, through the analysis of errors, was to be a diagnostic tool for use by the teachers.

Method

Subjects

The subjects sitting for the test were Year 8 students who had participated in the Mt. Druitt study in previous years as well as some additional students joining the study in 1986. The students attended either of the three Mt. Druitt high schools or the two comparison high schools numbering 603 in Mt. Druitt schools and 90 in the contrast schools, giving a total of 693 students.

Design

To fulfil the purpose listed above the instrument needed:

1) to provide a means of assessing student performance and detecting student weaknesses;
2) to have the potential to distinguish between the ability of the students in the different sites; and
3) to possess a range of items of varying difficulty, a number being items repeated from the 1985 testing, others being added at later testings to give a coverage of a range of difficulties appropriate to the age of the students.

Instrument

The instrument was compiled using items from the item bank of the ACER Maths Profile Series, the same bank used for the earlier 1985 test (Low, B. and Eltis, K. (in press)). The Maths Profile Series is a bank of questions which is designed to measure students' mathematical development throughout their schooling. This is achieved by way of a common scale of measurement. The items in the Series are grouped by their difficulty and the students are grouped according to their ability. The common scale is marked in equal units called "brytes" which indicate the difficulty of the item and the ability of the student for which it is a suitable item. Increasing brytes indicate increasing item difficulty and student ability. This item bank and scaling procedure provide a means of comparing the performances of students at any one time as well as providing a means of comparison over time.

The age norms (see Appendix 5), are based on experimental data collected by Collis (1975) and Wines (1975) and from ACER trial tests in a number of Victorian, New South Wales and Queensland schools. These norms are suggested as guides only because the trial schools, whilst possessing varying backgrounds, are in no way a representative sample of schools.

The items chosen for inclusion in the 1986 test were from four major areas; Measurement, Operations, Number and Space (Table 1).

Items covered the content and process areas of the four topics along the continuum of difficulty with bryte levels ranging from 42 to 54. These are lower than those suggested in the ACER Handbook for Year 8 students as the results from the 1985 maths testing indicated the suggested norms were too high (see Appendix 1-4 for the breakdown of the items over content area, processes and degree of difficulty and Appendix 5 for a description of the ACER Profile Series).

Table 1 **Classification of Items in ACER Maths Profile Series**

Major Area	Content Area
1. Measurement	a) Statistics, Money b) Perimeter, Area, Volume c) Measurement systems d) Time, Rates
2. Space	a) Basic concepts, Perception, Topology b) Location, Distance, Angular relations c) Shapes, Solids (properties & relations) d) Spatial transformations
3. Number	a) Natural numbers, Integers b) Indices, Logarithms c) Ratio, Percentage Rates d) Rational numbers
4. Operations	a) Small numbers b) Large numbers c) Pronumerals

The items are divided into three process areas which summarise the learning activity involved for each item. These are:

a) basic processes relating to knowledge of basic concepts, the ability to perform operations, to classify and to order objects;

b) understanding processes involving relevance, interpretation and communication; and

c) application processes in problem solving, reasoning and cognitive creativity.

Two tests of 30 questions were trialled on a group of 50 students from a school similar to the Mt. Druitt sub-sample and the results showed a spread consistent with the aims of the test.

As well the two tests were given to classroom teachers for examination and comment. This resulted in 19 questions being withdrawn for reasons such as; the relevant concepts had not yet been taught, for example the term "median" was unkown to the students; the topic had not been covered, e.g. percentages; the question was requiring too great a skill in reading comprehension. It was decided to reduce the second test to 24 questions due to the amount of time which would be required, according to the teachers, to answer the Space questions. Extra questions were re-submitted and the resulting tests were of 30 and 24 items respectively with the former comprising the Measurement and Operations items (15 of each) and the latter comprising the Number and Space items (12 of each). It was expected that both tests would be of 30 minutes duration in order to fit comfortably into the normal 40 minute mathematics period.

The tests (see Appendices 7 and 8 for the Test scripts) were administered on separate occasions, within one school week, by class teachers during March to all students of Year 8 who were in the study. Teachers had been instructed in the administration procedures and were given a copy of written instructions for administering the test (see Appendix 5).

The total number of students tested was 566 of whom 487 were from the Mt. Druitt schools sample and 79 from the contrast schools. The number of females was 295 and males, 269.

RESULTS

Mathematics Achievement

To determine whether any differences existed within and across the schools in the study a number of analyses were performed on the total sample, the subsamples (Mt. Druitt and Contrast schools), the individual schools and classes all of which were separated by gender for both the total score and subscores. The results are summarised in Table 2.

Table 2 **Differences in Mathematics Achievement**

Table 2A

Group	Number	Achievement Scores	
		Total Score mean	(max=54) s.d.
Total Mt Druitt Subsample	487	25.14	9.06
School 1	160	28.32	8.01
School 3	169	23.85	9.81
School 5	158	23.32	8.40
Total Contrast Subsample	79	38.21	9.11
School 2	46	36.51	10.14
School 4	33	40.64	6.84
Females	295	26.63	9.47
Males	269 *	27.41	10.86
TOTAL	566	27.00	10.11

Table 2B

Group	Number	Achievement Scores			
		Measurement (max=15)		Operations (max=15)	
		mean	s.d.	mean	s.d.
Total Mt Druitt Subsample	487	7.58	2.99	6.64	3.54
School 1	160	8.80	2.55	7.64	3.41
School 3	169	6.78	3.13	5.92	3.62
School 5	158	7.21	2.88	6.40	3.38
Total Contrast Subsample	79	10.88	3.04	11.14	3.49
School 2	46	10.47	3.55	10.19	3.97
School 4	33	11.49	2.02	12.49	2.08
Females	295	8.00	3.01	7.29	3.73
Males	269 *	8.12	3.40	7.25	4.03
TOTAL	566	8.05	3.20	7.28	3.86

Table 2C

Group	Number	Achievement Scores			
		Space (max=12)		Number (max=12)	
		mean	s.d.	mean	s.d.
Total Mt Druitt Subsample	487	6.47	2.41	4.45	2.37
School 1	160	6.86	2.19	5.03	2.25
School 3	169	6.57	2.50	4.49	2.50
School 5	158	5.99	2.46	3.73	2.04
Total Contrast Subsample	79	8.51	2.09	7.65	2.63
School 2	46	8.43	2.03	7.43	2.76
School 4	33	8.64	2.22	8.03	2.43
Females	295	6.60	2.35	4.74	2.49
Males	269 *	6.93	2.61	5.12	2.81
TOTAL	566	6.76	2.46	4.90	2.65

* 2 students did not indicate gender

Results reported in Table 2 indicate that, as expected from previous testing, the Mt. Druitt subsample of students achieved a significantly lower average score, compared to the contrast subsample, on all subscores and on the total score. The difference between the Mt. Druitt and Contrast school means of the total scores of approximately 13 is educationally important as well. The greatest difference in the mean total score of individual school groups peaked at 17 (between Schools 4 and 5), again educationally very important.

A series of t-tests was performed to determine significant differences between pairs of schools with respect to total scores and subscores. Table 4 indicates the pairs of schools between which there were significant differences on mean total scores.

Table 4 **Multiple Classification Analysis - School Group Differences on Total Scores**

	School 1	School 2	School 3	School 4	School 5
School 1					
School 2	*				
School 3	*	*			
School 4	*		*		
School 5	*	*		*	

* denotes pairs significantly different at .001 level

The results of this analysis on Total Scores indicate that:

1) both contrast schools were significantly different to the Mt. Druitt schools;
2) the contrast schools were significantly different to each other at $.02 < p < .05$;
3) Mt. Druitt 1 was significantly different to the other Mt. Druitt schools.

The results of the analysis on the Operations subscore (Table 5) indicate that:

1) the contrast schools were significantly different to the Mt. Druitt schools;
2) the contrast schools were significantly different to each other;
3) Mt. Druitt 1 was significantly different to the other Mt. Druitt schools.

Table 5 **Multiple Classification Analysis - School Group Difference on Operation Subscore**

	School 1	School 2	School 3	School 4	School 5
School 1					
School 2	*				
School 3	*	*			
School 4	*	*	*		
School 5	*	*		*	

* indicates pairs being significantly different at .001 level

The performance by Mt. Druitt school 1, in being significantly different (superior) to Mt. Druitt schools 3 and 5 in the Operations subscore and the Total score suggest that the Mt. Druitt subsample may not be homogeneous in maths performance. These results differ to those found in earlier, primary maths testing where the Mt. Druitt schools demonstrated greater homogeneity having no significant differences at .001 level (Houghton, 1981; Houghton and Low, 1982). This raises an important question - what are the features of, and what are the differences between, this school and the other two schools, all of which are from the same area, which could account for this result?

Progress in Mathematics Knowledge - Achievement "Gaps"

A growing divergence between the performances on standardised tests of disadvantaged youngsters and those of their non-disadvantaged peers has been frequently noted. This means that the longer the disadvantaged youngsters stay at school the further, relatively, they fall behind. In Table 6 are reported standard deviation unit differences; measures of relative performance. Whilst it is clear from the Table that across the primary years the Mt. Druitt students learning rate was somewhat better than the contrast schools' students (see the final column) a significant gap opened up with transition into high school. Fortunately the gap closes slightly in Year 8, indicating that, once again, the Mt. Druitt youngsters are learning at a slightly better rate than the contrast students. The increase in the achievement gap occurred with a change in maths testing style as well as with the primary school/high school transition. It would be fair to say that the high school maths tests (Years 7 and 8) are cognitively more demanding than those of earlier years. As could be suggested from the primary school performance, the Mt. Druitt students appear in high school to be somewhat slower in rate of intellectual development. Clearly though, the Year 8 results indicate they are now catching up as they did in primary school. The decrease in the standard deviation unit confirms that.

Table 6 **ACER Mathematics Achievement Results Expressed as Standard Deviation Unit Differences for Mt. Druitt Compared to Contrast School Children**

Year	Grade	Contrast n	Contrast $\bar{X}$	Contrast s.d.	Mt. Druitt n	Mt. Druitt $\bar{X}$	Standard Deviation Unit Difference*
1981	3	307	89.96	10.71	577	73.99	-1.49
1982	4	275	59.17	12.57	483	46.37	-1.02
1983	5	201	71.47	13.95	433	59.84	-0.83
1984	6	183	70.62	11.13	379	63.12	-0.68
1985	7	54	26.96	2.09	383	22.11	-2.32
1986	8	79	38.21	9.11	487	25.14	-1.44

* s.d. unit difference = $\frac{\bar{X}\text{ (Mt Druitt)} - \bar{X}\text{ (Contrast}}{\text{sd (Contrast)}}$

Table 6 is best read in two parts, the first from 1981 through 1984 and the second from 1985 onward. This break coincides with the move to high school and to a difference in approach to testing maths achievement.

Gender Differences

Throughout the primary school years (1981-1984) of the Mt. Druitt Study the results of the mathematics testing have shown that in the Mt. Druitt schools the females have achieved superior results on maths tests.. The reverse occurred at the Contrast schools. It was appropriate to examine whether this pattern is continuing into secondary schooling. Accordingly, for the 1986 results, an examination of each school's mean score and subscore by sex was undertaken.

The results are displayed in Table 7 indicating the relative performance of males and females in the test. Whilst the results for the Contrast schools maintain the superiority of the boys, of particular interest are the results for the Mt. Druitt subsample. The only significant differences in Mt. Druitt schools occurred for the Space subscore where in School 1 the males were significantly superior ($p = .001$) and in School 3 where the females were superior ($p = 0.28$). School 3 was the only school which maintained the superiority of females. The girls in this school were significantly better on one subtest, better on one other and equal to the boys on one other. The boys were better only on the measurement area. There is a distinct contrast to another Mt. Druitt school, number 1, where the pattern of male superiority on all but one measure is just as clear. How do these differences come about? Are they attributable to the method of teaching, the structure of the curriculum, the refusal of the females at one school to succumb to the male mathematics superiority stereotype or some other factor(s)? Investigation of the inconsistencies which are revealed by this analysis of the gender differences is warranted but such investigation is beyond the scope of this report.

Table 7 **Gender Differences in Performance in Subscores and Total Score**

	Mt. Druitt School			Contrast School	
	1	2	3	1	2
Measurement	*	=	√	√	√√
Operations	√	*	*	√	√
Space	√√	√	**	√	√
Number	√	√	=	√	√
Total	√	=	*	√	√

* indicates females superior
** indicates females significantly superior
√ indicates males superior
√√ indicates males significantly superior
= indicates no difference.

ERROR ANALYSIS

An analysis of the students' errors was undertaken to provide teachers with a means of identifying conceptual problems, computational problems and other error types with a view to their providing remediation. For each student a total score and four subscores were furnished to their mathematics class teachers as well as a

breakdown of each student's response, thereby providing a diagnostic tool for the teacher. These individual results are not included in this report for reasons of confidentiality. As well a breakdown by school, class and subscores was given to each teacher. These measures were designed to provide the teachers with as useful a diagnostic tool as possible.

The second aspect of the analysis of errors was the categorising of types of errors made by the students. In this analysis only those errors which represented a large proportion of the total error for a particular question were analysed. In some cases it was not possible to determine the strategy used by the students in making particular errors.

Error Strategies for Measurement Items (Questions 1-15)

There appear to be five types of errors which occurred in this subdivision of the Maths test:

1) Incorrect calculation: in this case the students applied the correct operation but made an error on computing, e.g Question 15 where the problem reduced to calculating 50-17 and students answered 23.

2) Interpretation of the given diagram: in this case the orientation of the student led to an incorrect response
   e.g. Q5.

3) Interpretation of the question: in this case the student is unable to isolate the problem
   e.g. Q13.

4) Knowledge of units of measurement: in this case the inability to place items in relative positions has led to error
   e.g. Q1.

5) Conceptual: in this case the error was through lack of knowledge of the particular idea being tested
   e.g. Q6 where the "acute" angle is asked for and a student responds by giving the "obtuse" angle.

Table 8 summarises these errors.

Table 8 **Error Analysis of Measurement Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of * Total Errors 1	2	3	4	5	No Response %
Units of measurement	1	81	18				99		1
	6	59	40				44	54	1
	7	58	40		55		40		2
	12	30	67	39	31		24		3
Rates, time	2	66	33	65		32			1
	3	68	31	47				34	1
	4	78	21	33			22		1
	9	28	70			44	33		2
	14	43	51	32				60	6
	15	33	64			57		22	3
Perimeter, Area, Volume	5	61	38		97				1
	11	47	49		91				4
	13	57	39			93			4
Statistics, Money	8	45	49	60					6
	10	51	41	31		61			4

** Does not include "No Response"

* Code for error strategy
1 : incorrect calculation
2 : interpretation of diagram
3 : interpretation of question
4 : knowledge of units
5 : conceptual

Error Strategies for Operation Items (Questions 16-30)

In this area seven error strategies were identified:

1. Inaccurate calculation : the correct operation was performed inaccurately and students obtained an answer that was one unit or one ten different to the correct answer
e.g. Q16 $15 \div 5 = 2$ (1 unit off).

2. Wrong operation or no operation : the answer given results from the wrong operator or is one of the given numbers in the question
e.g. Q27 $654 - 543 = \Delta - 654$
$\Delta = 1197$ suggests the numbers have been added.

3. Order of operations : the answer was arrived at by ignoring the correct procedure for evaluating
e.g. Q28 $(24 \div 6) \div 2 = \Delta \div (6 \div 2)$
$\Delta = 24$.

4. Zero misunderstanding : the student has thought that adding zero changes a number or multiplying by zero gives something
e.g. Q23 $w \times o = w$.

5. The student matches numbers or pronumerals in both expressions or ignores differences
   e.g. Q17 $8 \div 4 = 4 \div 8$.

6. The student ignores extra term and/or performs the calculation on the left hand side ignoring the right hand side
   e.g. Q18 $7 - 4 = 3$.

7. Unity misunderstanding : where multiplying by 1 gave something else or addition of 1 led to incorrect answer
   e.g. Q22 $v + 1 = w$.

Table 9 summarizes these operations errors.

Table 9 **Error Analysis of Operations Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of * Total Errors							No Response %
				1	2	3	4	5	6	7	
Small numbers	16	61	36	37				25	38		3
	17	38	59					63	19		3
	18	41	55					58	24		4
	26	33	60					76			7
	28	32	61			48					7
Large numbers	19	59	35	45				40			6
	20	63	34				87				3
	27	31	63					55	20		6
	29	46	45		34				46		9
	30	29	63					69			8
Pronumberals	21	66	30	20	24		65				4
	22	57	39		19					72	4
	23	56	39				77				5
	24	66	28	32				32			6
	25	51	44	63						31	5

** does not include "No Response"

* Code for error strategy

1. Inaccurate calculation
2. Wrong operation
3. Order of operation
4. Zero misunderstanding
5. Number matching
6. Ignores extra terms
7. Unity misunderstanding

Error Strategies for Space Items (Questions 31-42)

In this area four error types were identified:

1. Visualisation: in which case students did not see the "whole" of the problem or could not match the parts to the whole
   e.g. Question 35 - not seeing the "whole" of the problem
   Question 11 - not matching the parts to the whole.

2. Orientation: in which case students could not place themselves in the correct position to determine the correct response
   e.g. Question 9.

3. Topological: in which case students could not complete paths or patterns to obtain the correct response
   e.g. Question 6.

4. Conceptual: in which case students were unable to obtain the correct response because of incorrect matching of visual stimulus with concept
   e.g. Question 1 where response A and D were given because the lines are of different lengths.

Table 10 summarizes these errors.

Table 10 **Error Analysis of Space Items**

Objective	Item* No.	Success Rate	Error*** Rate %	Error strategy as % of ** Total Errors 1	2	3	4	No Response %
Perception, topology	34(4)	74	26		38	61		0.2
	36(6)	88	12			100		0.3
	40(10)	56	42			95		2.0
Location, distance	31(1)	59	40				98	1.0
Angular relations	33(3)	55	44		100			1.0
	42(12)	52	46		25		75	2.0
Shapes, solids	35(5)	43	56	98				1.0
	37(7)	55	45	100				0.2
	41(11)	32	64	72				4.0
Spatial transfer	32(2)	62	38	100				0.2
	38(8)	60	38	95				2.0
	39(9)	41	59		100			0.2

* The number in parenthesis indicates the number of the item in Test 2 whilst the other number is the item referred to throughout the report.

** Code for Error Strategy
1. Visualisation
2. Orientation
3. Topological
4. Conceptual

*** Does not include "No Response"

Error Strategies for Number Items (Questions 43-54)

Four general error strategies were defined for these items:

1. Incorrect operation: in which case the correct data was used with an incorrect operation to obtain an incorrect response
   e.g. Question 53 $\frac{210 \times 3}{7}$ instead of $\frac{210 \times 7}{3}$.

2. Incorrect calculation: in which the data was used correctly but the answer was incorrect by a certain factor or by a unit.
   e.g. Question 54 16.6 + 7.8 = 23.14
   Question 47 5 + 19 + 7 = 5 + 11 + 17.

3. Conceptual: in which the student could not obtain the correct solution because of lack of knowledge of terms or processes
   e.g. Question 43 4 x m = m + m + m + 4.

4. Interpretation of question: in which case the incorrect answer was obtained by the wrong reading of the question
   e.g. Question 54 where, in obtaining response B the student has interpreted the question to read "... risen to 16.6C" as opposed to "risen by 16.6C".

Table 11 summarizes these errors.

Table 11 **Error Analysis of Number Items**

Objective	Item No.*	Success Rate	Error Rate %***	Error strategy as % of Total Errors**				No Response %
				1	2	3	4	
Natural nos, integers	43(13)	60	38			98		2
	44(11)	54	43		90			3
	47(17)	38	57	43			35	5
	48(18)	36	60		22	70		4
	51(21)	41	54			92		5
Ratio, rates, percentage	52(22)	40	55	20		30	42	5
	53(23)	32	62	53		37		6
	45(15)	38	60		22			2
Rational numbers	46(16)	52	46		29	39		2
	49(19)	26	70				65	4
	50(20)	37	57			90		6
	54(24)	37	56	27	25		38	7

* The number in parenthesis indicates the number of the item in Test 2, whilst the other number is the item referred to throughout the report.

** Code for Error Strategy
1. Incorrect operation
2. Incorrect calculation
3. Conceptual
4. Interpretation

*** Does not include "No Response"

Discussion of Error Analysis

Before discussing these analyses generally it must be pointed out that certain assumptions are being made in obtaining these figures for the error analysis, not the least of which is that the students, in responding, were selecting the responses they felt most appropriate. It is impossible to isolate those cases where students have answered randomly. The other major assumption is that the tests were administered in such a way that no collusion was possible between students in making repsonses. (In fact, it is probable that either one or both of the above events did happen, however, it cannot be controlled.) The large number of poorly answered questions (see Table 11) may be due to the norms, as suggested by ACER, being unsuitable for students in Sydney. This statement refers, not only to the Mt. Druitt schools where the expectation is for poorer results, but to the Contrast schools where the results do not match those that would be expected from this relatively able group. The ACER Handbook states that the age norms given are only to be used as guides and that it may be necessary to develop "local" norms. This warning is reinforced when it is noticed that of the thirty-three schools trialled by ACER in developing the test, only four were New South Wales schools. For the Number test, Ibrox Park Boys High School and North Sydney Girls High School (a selective school) were used while for the Measurement Test Maroubra Bay High School and Pittwater High School students' responses were included. There are more than three hundred high schools in New South Wales. A sample of four probably does not represent a reliable sample.

The items in the Mt. Druitt Longitudinal Study test were selected using these "high" norms and the concomitant level of difficulty of items may have affected the attitude of the students towards the test.

Whilst there was variation between school groups with respect to answers to particular questions (see Table 12 for a summary of the most poorly answered questions) there are some generalisations that may be made. Firstly, there was less success for questions in the Operation area which involved large numbers. Secondly, there was a propensity for students to respond to questions such as Question 17: $8 - 4 = \Delta - 8$ by replacing Δ by the missing number from the left hand side. In this particular case, the response $\Delta = 4$ was given by 39 percent of students whilst the correct response was given by 38 percent of the students. Thirdly, whilst 63 percent responded correctly to Question 20: $769 \times 0 = \Delta$ only 56 percent did so for Question 23: $w \times o = \Delta$, thereby suggesting that many students have difficulty with the concept of zero and this is compounded when pronumerals are introduced.

A further major cause of errors may be associated with the student's inability to analyse the question before adopting problem solving techniques, if they possess them. This occurred largely with items in the Number and Measurement categories where questions were not read correctly. It must be added that the researchers are cognisant of the lower reading levels at the Mt. Druitt high schools and efforts were made to include questions which were not too difficult to read. At the same time, however, it must be pointed out that mathematics involves more than performing operations on numbers and that reading is an integral part of problem solving in mathematics.

Table 12 suggests that particular school groups are weaker in particular topic areas. For example School 3 had the lowest percentage of correct responses on the four badly answered Measurement questions whilst School 5 had the lowest percentage of correct responses on the nine badly answered Number questions. These results may be due to curricula emphases or programming of the subject.

The latter reason should be able to be discounted as the class teachers examined the test and items were withdrawn if subject matter had not been dealt with. Further analysis of this matter would be necessary before any firm conclusions may be drawn. However, the information should provide class teachers with the stimulus for critical appraisal of teaching methods.

Table 12 **Summary of School Group Errors**

		Percentage of Students Correct*					
Topic/Question		Total	School 1	School 2	School 3	School 4	School 5
Measurement	9	28	28	49	24	32	25
	11	47	51	66	36	79	42
	12	30	30	60	22	65	25
	15	33	31	53	27	62	29
Operations	17	38	39	68	34	74	26
	18	41	42	66	38	74	29
	26	33	31	63	27	76	24
	27	31	32	42	23	68	27
	28	32	23	51	32	71	28
	29	46	52	55	32	88	45
	30	29	27	55	21	74	23
Space	35	42	43	51	46	56	34
	39	41	40	55	38	50	39
	41	32	31	43	30	50	27
Number	45	38	39	60	36	56	29
	47	38	37	62	39	41	29
	48	36	34	49	36	53	29
	49	26	23	45	28	56	17
	50	37	38	57	33	68	27
	51	41	45	51	44	65	26
	52	40	40	72	39	76	24
	53	32	30	53	29	53	29
	54	37	35	74	32	74	26

* applies to most poorly answered questions (less than 50% correct on total)

The Space items provided the highest level of response to questions and at the same time provided the greatest proportion of relatively well answered questions. Whilst no conclusion can be drawn from this, the phenomenon suggests research questions such as:

"is it because of the visual nature of the questions?"

"is it because the students are more at ease with questions they can physically manipulate?"

As the topic area Space was not treated in the 1985 testing, it is not possible to comment on any pattern. However, for the testing proposed for 1987 it will be worthwhile to follow up to determine whether the same phenomenon occurs.

Interpretation of the "No Response" Category

In attempting to explain the total lack of a response to a specific item, which ranged from less than one percent to nine percent of the total responses for a particular item, it is not possible to determine whether the students could not or would not answer the question. The fact that the number of no responses increases towards the end of both tests may suggest that test fatigue could be in evidence. Observation of students during the tests and/or follow up interviews would enable more conclusive statements.

Summary of Findings

1. There are significant differences in achievement between the two subsamples and between schools within the subsamples on both total score and subscores. The Mt. Druitt and Contrast subsamples were significantly different on all subscores and total scores. Mt. Druitt School 1 was seen to be significantly different to the other two schools in the Mt. Druitt subsample.

2. The analysis of errors revealed common error strategies within and across the subsamples. The major factors affecting errors appear to be: the interpretation of the question, the relative size of numbers and the position of the symbol whose value was to be determined.

ERROR ANALYSIS AND IMPLICATIONS FOR TEACHING : SOME POSSIBLE INTERVENTIONS

Following the responses made by students to the questions in the 1986 Mathematics test, what follows are some suggestions for general and specific strategies which may be of assistance in dealing with the errors. It is hoped that these suggestions might be useful and feedback to the research team from teachers will be sought. Several references which deal with error strategies and which may provide more help are identified.

Language and Mathematics

A concern of many of the teachers in the study was that some of the test items were too "wordy" and were not testing Mathematics and hence should be omitted. However, rather than avoiding these types of questions, it is important that students should come to grips with them. It is essential that language skills necessary to mathematics be developed. The Cockcroft Report (1982:91) suggests this would be best achieved through clear, unambiguous explanation, joint discussion and by encouraging students to talk and write about their investigations.

There are two major aspects of language relevant to this test. Firstly, there is the search for verbal cues made by students in an attempt to link an operation with the written word. Unfortunately, this search often leads to errors because of different operations being possible for the same word or because of the student wrongly interpreting the cue word.

An example of the first case would be the following situation:

1. Peter has \$5 and Jenny has \$3 more than Peter. How much money does Jenny have?

2. Peter has \$5 and Jenny has \$8. How much more does Jenny have?

Here the word "more" has associated with it two different operations. It is essential that students learn to determine the context of the particular cue word.

An example of the second case occurs in Question 54 where many students interpreted the word "by" as "to" and subtracted the two temperatures instead of adding. Similar problems occur in cases such as "a fall of" compared with "a fall to", "to 12th place" compared with "from 12th place", "a bearing of A from B" compared with "a bearing from A to B".

The second aspect of language is that, upon finding the cue words, it is important that students refer back to the question as a whole.

e.g.1. In Question 9 the majority of students having found the cue words "name of the month" and "starts with A" did not relate this information to the word "before". Hence 32 percent of students obtained "August" as the answer whilst 28 percent had the correct response.

e.g.2. In Question 15, the word "remaining" was not correctly identified and 38 percent of students gave as their answer the amount of fuel used (33% were correct).

Thus greater exposure to, and discussion of, questions involving the use of language is recommended whilst the minimisation of language usage in questions is discouraged.

Rational Numbers

For many students the concept of a fraction is initially bound to sharing and is concerned with the relationship between "a part" and "the whole". Thus in Question 49, where 48 percent of students had 1/3 as their answer (26% had the correct response of 1/6) this conceptualisation of a fraction was being used with the "half of the apple" representing the "whole". This illustrates that students encounter problems with fractions when the problem involves a part/part relationship.

Whilst students should have experienced the use of concrete materials in learning about fractions prior to Year 8 the learning appears not to have been effective. It is recommended that students actually see (so that they can later visualise and generalise) a half of the apple being divided into three parts and these pieces being related to the whole apple. This approach also reinforces the relative size of fractions and their relative position on the number line as was tested in Question 50 where from the number line given

students were asked which of 1/2, 3/4, 4/5, 1/8 is a point in the shaded section (37% were correct whilst 28% chose 3/4).

An approach to teaching the relationship is to actually generate the misconception, that is, that half an apple divided amongst three people gives each person a third of an apple and discuss the implications of this, using an apple (or whatever is appropriate) if necessary and then resolve the problem, relying heavily on student participation. (This requires considerable self-control from teachers who have an almost subconscious desire to put words into the mouths of students.) This so-called Conflict Approach (Bell, 1984) can be very effective in resolving misconceptions.

A further example of using this approach, cited in Bell (1984) can be useful with students who consider decimals to be two separate integers separated by a ".". A "conflict" approach would be to have students count orally 0.3, 0.6, 0.9 ... etc. Those who have the mentioned misconception would continue 0.12, 0.15 and so on. Having students do the "counting" on a number line or calculator leads to a confrontation with the error and, hopefully, to a resolution.

Operations

Many of the errors which occurred in this section stemmed from the over-generalisation of a previously learned rule; e.g. in Questions 17, 18 and 27 many students (39%, 35% and 38% respectively) over-generalised the commutative law of addition and multiplication to subtraction and division. To overcome this, the giving of varied examples and non-examples of the rule should aid in countering this misconception. In other words, rather than have students witness that $7 + 5 = 5 + 7$ and $7 - 5 \neq 5 - 7$, students should also witness that, for example, $615.95 + 37.6 = 37.6 + 615.95$ and $615.95 - 37.6 \neq 37.6 - 615.95$ and that $11.1 \div 8.74 \neq 8.74 \div 11.1$. Too often these types of rules are introduced using "easy" or "simple" examples whereas varying the numbers could lead to correct generalisations of the rules. This would then prevent the increase in errors when one considers Questions 26 and 30 where exactly the same rule is applied yet an extra four percent of students were incorrect in the question involving large numbers.

The over-generalisation problem appears to have occurred with respect to addition and multiplication by 0 and 1, e.g.

Question 20	24% of students responded that $769 \times 0 = 769$
Question 23	29% of students responded that $w \times 0 = w$
Question 22	27% of students responded that $v + 1 = v$
Question 21	18% of students responded that $g + 0 = 0$, and only 50% of students responded that $y \times 1 = y$ (Question 25).

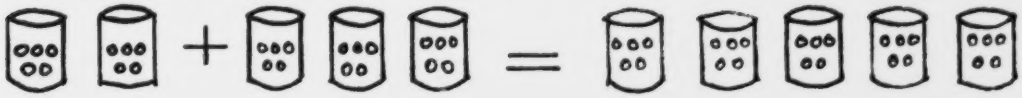
These errors, which occurred more frequently but not exclusively in the lower ability classes, could also be attacked using the conflict approach, illustrating the incorrectness of the student's answers and then the correct response being reinforced using examples and non-examples.

That more students were incorrect when a pronumeral was introduced as is the case of Questions 20 and 23 suggests that the concept of pronumerals is not clearly understood. This misunderstanding is further emphasized in Question 22 in which students responded that $v + 1 = w$. It is important that students do not interpret a pronumeral as an object, an occurrence which, unfortunately, tends to be reinforced by operations such as addition of monomials; e.g. $2x + 3x$ and $2a + 3b$. Students should never be told, as they too often are, for the latter example: "since you cannot add tables and chairs, you cannot add a's and b's". This type of justification, as well as being mathematically unsound, has inherent problems. It does not address the concept of a pronumeral.

A pronumeral does not represent a "table" or a "chair", it represents a number. A more practical way of illustrating this would be to use counters and cups along the following lines, noting that the teacher has emphasised that "x" represents the same number in each separate situation.

For 2x + 3x, the teacher arbitrarily chooses a number to represent x, say 5, without telling the students, and places this number of counters into each of five identical cups.

Hence 2x + 3x becomes:



The students then count the number of counters in the total and see that the answer is not five cups but five lots of the number inside each cup. It is important that the students see the counters at the end to reinforce the idea that x represents a number. (This method can be extended to make the teaching of solutions of equations of the type $5n - 6 = 3n + 12$ more meaningful and comprehensible.)

It is then a relatively simple matter to illustrate why 2a and 3b cannot be added, assuming a and b are different numbers.

The second problem with treating pronumerals as objects is that it leads to confusion in the minds of the students when multiplication of monomials is introduced. Having been told that 3a and 2b cannot be added because "you cannot add tables and chairs" the student now finds that he can multiply tables and chairs! This inconsistency and confusion is eliminated quite simply by always referring to pronumerals as numbers.

Spatial Questions

The major reason for errors in this aspect of the test probably stems from a lack of experience with manipulation of shapes and objects. However, the new Year 7 and Year 8 syllabus is addressing this matter. The DIME range of materials in both 3-D solids and Algebra are recommended for facilitating spatial perception and developing algebraic concepts in a readily understandable and enjoyable way.

Some Concluding Remarks

Some general observations may be made concerning the analysis of errors and learning interventions.

Firstly, as the Cockcroft Report (1982) and Hart, K.M. (1984) suggest, teachers of mathematics need to analyse the processes that students use in solving problems so that misconceptions can be identified and corrected. This can be done by examining scripts and/or by small or large group discussion of the questions and students' suggestions for their solutions. This process, whilst far more demanding and time consuming, is of much greater importance than that of determining whether a response is correct or incorrect.

Secondly, for developing understanding of mathematical concepts there is little to be gained from giving students a set of problems involving the same concept with different numbers in different contexts (as is the case in most mathematics text books). It is preferable to concentrate on changes of the structure in a fixed context or on the transfer of structure to different contexts (Bell, 1984). This can be done by asking pupils to match questions of the same type to write questions of a similar type but with a variation in context, or to provide a stimulus and have students compose questions ,

e.g. the calculation of "speed" using $S = \frac{D}{T}$ is the same type of question as calculating the "price" using $P = \frac{\text{Total Cost}}{\text{Number}}$

Most textbooks treat these two processes under different headings in different parts of the book when they are clearly related. Teaching generic principles rather than merely aiding fluency is to be preferred as an instructional strategy.

Finally, it is necessary not to underestimate or underutilise the presage skills that students possess, nor to ignore what might appear as "ad hoc" methods in solving problems. Research indicates that in solving problems, primary and junior secondary students rarely use formal methods of solution. They do not identify the type of problem and apply the method of solution (Cockcroft, 1982; Hart, 1984; Bell, 1984). The implication is that problems should be developed from ones with which students can cope to more complex ones which do not involve steps which require memorisation or which lead to complicated equations. There is perhaps much truth in what Hart (1984:83) states when she says:

> that the best time to teach a child a new skill is when it is needed because the child is faced with a situation that cannot be dealt with using previously employed methods ... the (children) would retain and use "child methods" as long as they were adequate and more formalised procedures ... (are not used) until seen as essential.

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These references, with the exception of the Cockcroft Report, provide many illustrations of types of errors made by students and suggest ways of addressing the particular errors. As well as this, models of intervention are proposed which could provide the stimulus for developing on-going strategies in attacking students' misconceptions.

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Appendix 1

Breakdown of Measurement Items

Test Question	Test bank item/test	Brytes*	Content Area	Process Area
1	2/1	44	Systems	Basic
2	1/2	47	Time, rates	Application
3	6/1	49	Time, rates	Basic
4	12/1	48	Systems	Understanding
5	7/1	49	Area	Understanding
6	8/1	49	Systems	Basic
7	2/2	49	Systems	Basic
8	9/1	50	Statistics, Money	Application
9	4/2	51	Time, rates	Understanding
10	6/2	52	Statistics, Money	Application
11	11/1	52	Perimeter	Basic
12	3/2	51	Systems	Understanding
13	15/1	53	Area	Understanding
14	14/1	52	Time, rates	Application
15	21/1	53	Time, rates	Application

* a "bryte" is a metric suggested by Wright and Douglass (1975)

Appendix 2

Breakdown of Operations Items

Test Question	Test bank item/test	Brytes	Content
16	13	47	Small
17	16	50	Small
18	15	51	Small
19	28	42	Large
20	30	48	Large
21	47	45	Pronumerals
22	49	51	Pronumerals
23	50	49	Pronumerals
24	48	43	Pronumerals
25	44	47	Pronumerals
26	14	52	Small
27	35	52	Large
28	18	53	Small
29	33	53	Large
30	34	54	Large

Appendix 3

Breakdown of Space Items

Test Question	Test bank item/test	Brytes	Content Area	Process Area
31	1/2	44	Location	Basic
32	13/1	47	Spatial	Basic
33	14/1	47	Shapes	Understanding
34	4/2	47	Spatial	Understanding
35	3/3	49	Shapes	Basic
36	6/2	49	Perception	Basic
37	19/1	49	Shapes	Basic
38	25/1	50	Spatial	Basic
39	23/1	49	Spatial	Basic
40	17/2	51	Perception	Basic
41	12/3	53	Shapes	Application
42	2/4	42	Shapes	Application

Appendix 4

Breakdown of Number Items

Test Question	Test bank item/test	Brytes	Content Area	Process Area
43	4/1	47	Rational nos.	Basic
44	3/2	48	Ratio	Application
45	7/1	48	Rational nos.	Application
46	8/1	49	Rational nos.	Understanding
47	11/1	49	Natural nos.	Application
48	5/2	49	Natural nos.	Understanding
49	16/1	51	Rational nos.	Understanding
50	1/3	51	Rational nos.	Basic
51	19/1	52	Natural nos.	Basic
52	8/2	51	Natural nos.	Basic
53	20/1	52	Ratio	Understanding
54	4/2	49	Rational nos.	Application

The A.C.E.R. Maths Profile Series...

This series has been designed to provide a flexible system for monitoring students' mathematical development throughout their schooling.

Tests provided in the series use a measurement scale based on the Rasch approach. The Rasch scale provides a common measurement scale on which student ability and item difficulty are simultaneously calibrated, independently of any external characteristics of the students tested or the items answered, except those relevant to their interaction (ie. to the latent trait being analysed).The Rasch model of measurement of any trait is based on the idea that the odds of success by a person ona particular task depend on the interaction between the ability of the person and the difficulty of the task.eg. when a person's ability is greater than the task difficulty, then the odds of success are high. For Rasch scales the concepts of ability and difficulty are defined in terms of this concept of the relative odds of success which must be consistent across a variety of interactions between people and tasks.

The common scale used in the ACER Maths Profile Series is referred to as the MAPS Scale.This scale provides a measure of the latent variable which all the tests in the series have in common...ie. mathematical development. Items used in the series are located by their MAPS difficulty and students by their MAPS ability.The items have been located in their relative positions on the MAPS scale according to the degree of mathematical challenge that they offer students. These item measures have been called MAPS difficulties. For each set of items combined into a test, the marks obtained by students reflect the different levels of mathematical development elicited by the items offered. These total test scores can easily be converted into student measures on the common scale, thus giving the students' MAPS abilities.

The MAPS scale is marked in equal units called "brytes". Increasing brytes indicate increasing MAPS ability of students and increasing MAPS ability of items.

How can you use test results???

1) Achievement rankings and diagnostic profiles...THe common scale for all tests allows one to compare student MAPS abilities. This enables achievement ranking or grading across a group of students. Another purpose is to draw up profiles for each student in the class. This could be done for different content areas and assist in determining the relative strengths and weaknesses of a student. Profiles across time may also be useful.

2) Criterion-referenced evaluation and relative mastery...The Rasch measurement approach allows one to assess the likelihood of a student witha known MAPS ability

of being able to correctly answer any item of known MAPS difficulty. Following from this, mastery levels have been devised quoted in terms of percentage success over a group of items of representing a desired objective. Criterion levels of 100%, 90% and sometimes 75% are usually specified. Where student MAPS ability equals item MAPS difficulty the even chance of success can be interpreted as 50% mastery. As MAPS ability increases relative to MAPS difficulty so does the corresponding mastery level; and conversely as MAPS ability declines relative to MAPS difficulty so do expected mastery levels.The potential for interpretation at a variety of levels makes it possible to assess a student's status in relation to a sequence of learning objectives and to identify those objectives for which continued learning activities are most likey to be of most benefit to a student. Theory suggests that most suitable learning activities are likely to be reflected by items having MAPS difficulties within the range of -5 brytes and to +5 brytes of each student's MAPS ability.

3) Cognitive levels and their implication for student learning...the MAPS scale has been approximated to the Piagetian stages. 25% to 90% mastery ranges at each cognitive stage follow the following pattern:

Cognitive stage:	25% to 90% mastery
Early concrete operational	30 to 45 brytes
Late concrete operational	40 to 55 brytes
Concrete generalization (early formal)	50 to 65 brytes
Late formal operational	6 to 75 brytes

Approximate age/school 'norms' are as follows (but these may vary according to localty and mathematical background of the student):

Mean MAPS ability of students for ages 7 to 17 years

Age	Related years of schooling	Mean MAPS ability(nearest bryte)
7 years	2	29
8 years	3	37
9 years	4	44
10 years	5	47
11 years	6	49
12years	7	5
13 years	8	51
14 years	9	53
15 years	10	55
16 years	11	58
17 years	12	65

The ACER Maths Profile Test for Year 8: Mount Druitt Longitudinal Study.

This test consists of 60 items designed to monitor students' development in four separate aspects of mathematics.The items in each section have been chosen from a longer test from the ACER Mathematics Profile Sreies. As the four samples of items have been selected to represent the original test, the results from administering each sample can be used to give a profile of attainment in the four areas.

The four areas of mathematics covered in the test are:
operations;
measurement;
numbers;
space.

The first section of the test has 15 items from the ACER Maths Profile Series-Operations Test.The Operations test was designed to assess students' ability to handle familiar operations in the real number field. Five items involve operations upon small numbers, five involve operations upon large numbers and five have operations involoving pronumerals.

The second section has 15 items from the Measurement Test. This test was designed to monitor students' development in those aspects of maths pertaining to measurement and mensuration. Four items involve statistics or money, three involve area, volume or perimeter, four involve measurement systems and four involve time or rates.

The third section has 15 items from the Space Test. This test was designed to assess students' spatial development, considering both traditional topics and some of the perceptual aspects which underlie these topics. Three items involve basic concepts, perception or topology, four involve location, distance or angular relations, six involve shapes or solids including properties and relations and four involve spatial transformations.

The fourth section has 15 items from the Number Test. This test was designed to monitor students' development with regard to number concepts, properties and relationships. Four items involve natural numbers or integers, two involve indices or logarithms, three involve ratio, percentages or rates and six involve rational numbers.

General:

1) The ACER Mathematics Profile Series consists of two parts which are to be administered at different times. The first part consists of 30 items and the second, of 24 items. Both tests require about 30 minutes actual testing time. An additional 10 minutes should be allowed for introductory procedures.

2) There are practice questions on the second page of both test booklets. The prime function of these is to introduce students to the method of answering. The name and significance of Δ may vary according to year level.It is suggested that for up to Year 8, Δ could be discussed as the triangle or the frame in which the numeral can be written. If students are familiar with the concept of pronumeral, Δ could be discussed as delta and the mathematical sentence as an equation.

3) During administration the sections enclosed in boxes are to be read aloud. However the text can be adapted to better suit the needs of their particular students.

4) Students should have working paper available so that test booklets are not spoiled.

Testing:

1) Direct students to fill in the section of the answer sheet with school, name, class age and birth date.

2) Direct students to the instructions on the front page of the test booklet and then to the continuing instructions on the second page. Depending on which is more appropriate for your particular class, either: (a) ask students to silently read the directions up until the practice questions, or (b) ask students to follow silently as you read the directions aloud.

3) The purpose of the practice questions is to ensure that students know how to recordanswers to multiple choice questions. Please do not show students how to perform the mathematical operations necessary for answering the questions.

Look at practice question P1. It says '5 x 17 = 5 x Δ'. Put your hand up if you know the answer.

Obtain the correct answer from a student.

Yes, to make the sentence true, Δ stands for 17. So the letter C has been marked on your answer sheet beside P1.

Students can be referred to their answer sheets to see how the answer to P1 is recorded.

4 Now try practice question P2 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, B is the correct answer, so you should have marked the letter B on your answer sheet beside P2.

Teachers may wish to demonstrate this on the chalkboard and check that students have placed the correct letter in the correct place.

5 Now try practice question P3 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, A is the correct answer, so you should have marked the letter A on your answer sheet beside P3.

6 Now try practice question P4 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, A is the correct answer so you should have marked the letter A on your answer sheet beside P4.

7 Are there any questions about this method of answering?

Answer any questions concerned with the method of answering.

8 You will have 30 minutes (or whatever time is appropriate) for this part. Work as carefully as possible but do not spend too much time on any one question. If you cannot answer a question after thinking about it, leave it and go on to the next one. You may have time to come back when you have finished the rest of the part. If you finish before time, check your answers, then close your booklet and wait quietly. Are there any questions?

If no further questions, say:

9 Turn over to the next page and start work now.

Record the time and check that all students have turned to the correct page.

10 After 30 minutes (or whatever time is appropriate), say:

Finish the question you are working on, then close your booklets.

11 As answer sheets are collected, check that the information is correct and entered clearly.

12 Check that all test booklets and answer sheets have been handed in before concluding the testing activity.

13. For the second test, please use this same model for administration.

The correct answers for the four practice questions are

P1	C
P2	B
P3	C
P4	D

MACQUARIE UNIVERSITY

SCHOOL OF EDUCATION

MT DRUITT LONGITUDINAL STUDY

MATHEMATICS TEST*

THERE ARE 30 QUESTIONS IN THIS MATHEMATICS TEST. ALTHOUGH SOME QUESTIONS MAY NOT BE EXACTLY THE SAME AS THE WORK YOU DO IN CLASS, YOU SHOULD TRY TO WORK OUT THE CORRECT ANSWER.

PLEASE ANSWER EACH QUESTION BY CIRCLING ONE OF THE LETTERS ON THE ANSWER SHEET. PLEASE DO NOT MAKE ANY MARKS IN THIS BOOKLET.

THANK YOU FOR YOUR HELP.

* Items selected from ACER Mathematics Profile Series.

MATHEMATICS TEST

Practice Sheet

This test contains 30 questions.

In each question alternative answers are shown by the letters A, B, C, D or E. Mark your answer with a circle on the answer sheet.

The following practice questions will show you how to answer the questions on the text. When you see △ in a question you should choose the alternative to replace △ and make the sentence true.

Practice Questions

P1

$5 \times 17 = 5 \times \triangle$

A $\frac{17}{5}$ (C 17)

B 5 D 85

P2

$p + q = q + \triangle$

A r C $-p$

(B p) D $p - 2q$

P3

The measure of the shaded region in this diagram is called the

(A area.) C circumference.

B perimeter. D volume.

Since area is the correct answer, the corresponding letter A has been marked on the answer sheet.

P4

Which one of the following measures the longest distance?

(A 1 kilometre) C 100 millimetres

B 10 metres D 1000 centimetres

Please wait until you are told to begin the test.

1. If the length of a school room measures 5.5 units, which one of the following is the most likely unit being used?

A millimetre **C** metre
B centimetre **D** kilometre

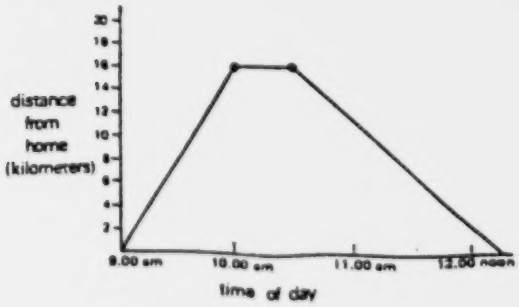
2. An empty tank is filled from a hose. If the water flows from the hose at the rate of 300 litres per hour and the tank holds 200 litres, how long will it take to fill?

A 30 minutes **C** 60 minutes
B 40 minutes **D** 90 minutes

3. It takes John 5 hours to travel 280 kilometres. Which one of the following is his average speed for the journey?

A 28 kilometres per hour
B 56 kilometres per hour
C 275 kilometres per hour
D 280 kilometres per hour

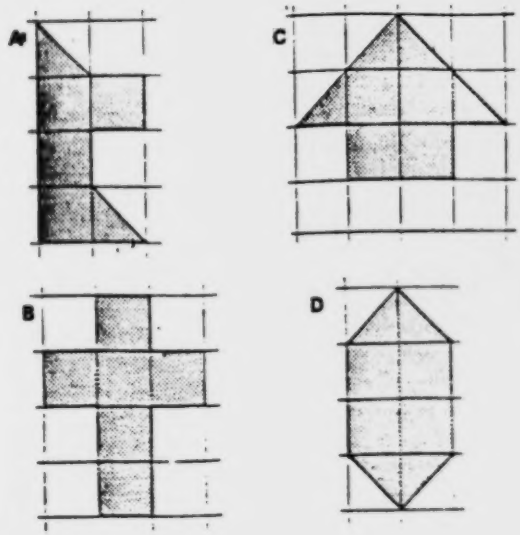
4. Jane made a cycle trip from home to visit her grandmother and back home again. She cycled directly to her grandmother's where she rested, and then cycled directly home. The graph shows her distance from home during the trip.



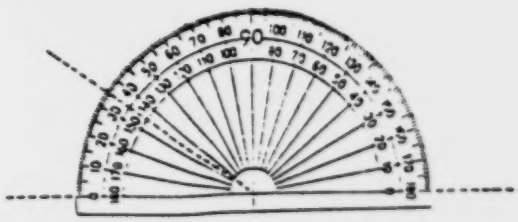
Which one of the following is the distance from Jane's home to her grandmother's home?

A 3 km **C** 9 km
B 8 km **D** 16 km

5. Which one of the following shaded shapes has the **least** area?



6. A student is measuring this acute angle with this protractor.



Which one of the following is the correct reading?

A 155° **C** 45°
B 145° **D** 35°

7. Which one of the following is the best estimate of the size of this angle?

A 150° **C** 90°
B 110° **D** 75°

8. Jane asked Maria to buy her some jewellery, costing 10 000 lire, in Florence. If one Australian dollar was equivalent to 850 lire, how many Australian dollars does Jane need to give Maria to buy the jewellery?

A $12 **C** $90
B $85 **D** $117

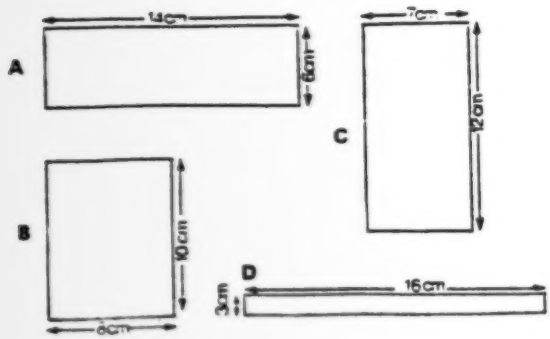
9. Sam's birthday occurs in a month which has 31 days. The name of the month before Sam's birthday starts with an 'A'. Sam has his birthday during

A March. **C** July.
B May **D** August.
E September.

10. In a sale, all model aeroplane kits are reduced by 10%. The two kits Peter wants are usually $1.35 and 65 cents.
How much will Peter save if he buys these two kits in the sale?

A 10 cents **C** $1.80
B 20 cents **D** $1.90

11. Which one of the following rectangles has the longest perimeter?

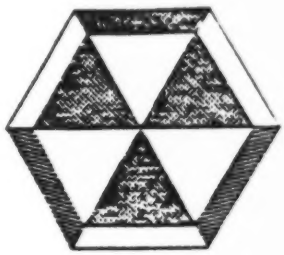


12. In this diagram, one unit on the scale represents 12 kilometres.

The distance from X to Y is nearest to

A 2.5 km **C** 24 km
B 12.5 km **D** 30 km

13. Three students made the following statements about this diagram.



Jim said, 'For an accurate measure of area, the diagram has been divided so you can count units of comparable size'.
Jane said, 'There are two different measuring units in this diagram which can be compared'.
John said, 'Exactly half of the diagram is shaded'.

Which of these students was correct?

A Jim only **C** Jim and Jane
B Jane only **D** Jim, Jane, and John

14. The 'year' and 'day' are divisions of time based on the earth travelling once around the sun (1 year) and revolving about 365 times on its own axis (365 days). The 'week' is now taken to be seven days, but in ancient times, the length of a week varied in different civilisations.

Suppose we had inherited the ancient Assyrian convention that 'a week = 6 days'. The number of weeks in a year would be closest to

A 40. **C** 60.
B 50. **D** 70.

15. A car's fuel tank contains 50 litres of petrol, and the car uses one litre of petrol every 10 kilometres. If the car travelled 170 kilometres, the quantity of fuel remaining would be

A 10 litres. **C** 23 litres.
B 17 litres. **D** 33 litres.

16. $15 \div 5 = 30 \div \triangle$

A 15 C 3

B 10 D 2

17. $8 \div 4 = \triangle \div 8$

A 32 C 4

B 16 D 2

18. $7 - 4 = \triangle - 7$

A 11 C 4

B 10 D 3

19. $(89 - 67) \div 33 = \triangle \div (67 \div 33)$

A 122 C -89

B 89 D -156

20. $769 \times 0 = \triangle$

A 7690 C 769

B 770 D 0

21. $g + 0 = \triangle$

A h C 0

B g D $-g$

22. $v + 1 = \triangle$

A w C $v - 1$

B v D $v + 1$

23. $w \times 0 = \triangle$

A w C 0

B 1 D $w \div 0$

24. $(h + j) \div k = \triangle + (j + k)$

A h C $h \div k$

B k D $h - k$

25. $y \times 1 = \triangle$

A $y + 1$ C 1

B y D $\frac{1}{y}$

26. $4 + 5 = (4 + 6) + (5 + \triangle)$

A 6 C -1

B 4 D -6

27. $654 - 543 = \triangle - 654$

A 1197 C 543

B 765 D 111

28. $(24 \div 6) \div 2 = \triangle \div (6 \div 2)$

A 24 C 6

B 12 D 4

29. $240 \div 15 = 480 \div \triangle$

A 30 C $\frac{15}{2}$

B 16 D $\frac{2}{15}$

30. $123 + 456 = (123 + 789) + (456 + \triangle)$

A 789 C -123

B 123 D -789

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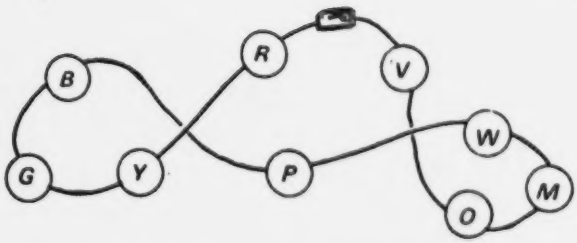
Practice Questions

P_1

Some words and numerals have the same meaning if they are read upside down. SWIMS is an example of such a word.
Which one of the following numerals means the same number, if we read it upside down?

A 8181
B 8108
C 609
D 6996

P_2



If the bead labelled V was threaded first, what was the order for threading the **rest** of the beads?

A W, M, O, P, Y, G, B, R
B W, M, O, P, B, G, Y, R
C O, M, W, P, Y, G, B, R
D O, M, W, P, B, G, Y, R

P_3

A person's average walking speed is 6 kilometres per hour. Which one of the following is the distance the person would travel in 1 hour?

A 2 km
B 3 km
C 6 km
D 12 km
E 24 km

P_4

Which one of the following gives 659 rounded to the nearest hundred?

A 600
B 650
C 660
D 700

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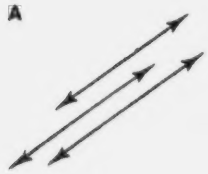
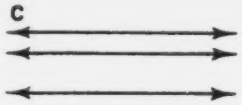
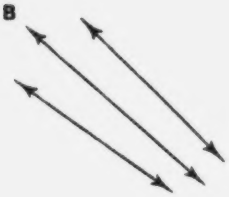
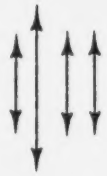
1) Which one of the following is a group of lines which are not parallel?

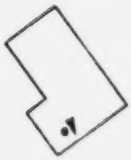
A 

C 

B 

D 

2) Which one of the following shows the same face as card I?

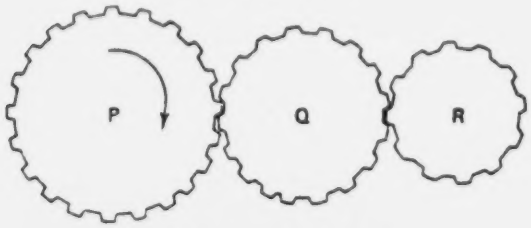
A 

C

B

D

3) P, Q, and R represent three cogwheels and P is turning in a clockwise direction



Which one of the following is true?

A Q turns clockwise
B R turns clockwise
C Q and R turn clockwise
D Q and R turn anticlockwise

4) This pattern has been woven right through the fabric of a flag which is blowing in a strong wind.

Which one of the following shows the flag when the wind changes to the opposite direction?

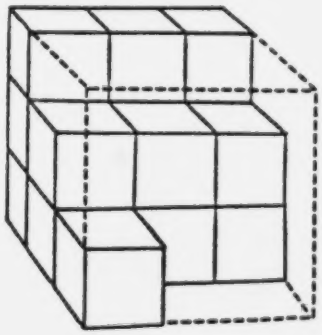
A

C

B

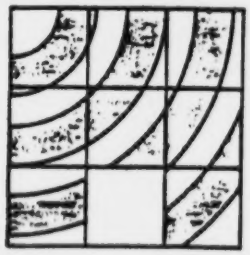
D

5) How many more small cubes would be needed to complete the larger cube, as outlined by the dotted lines?



A 8
B 9
C 11
D 14

6) Which one of the following pieces will fit into the blank space and complete the design?



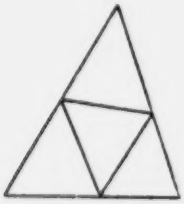
A

C

B

D

7) How many triangles are there in this figure?



A 3

B 4

C 5

D 6

8) Some words, like **MUM**, look the same if we read them in a mirror placed vertically beside them.

Which one of the following would represent the same number if read in a mirror placed beside it?

A 30103

B 696

C 8181

D 8008

9) This clown's head was drawn on the outside of a balloon.

Which one of the following shows how the head would appear if it could be viewed from inside the balloon?

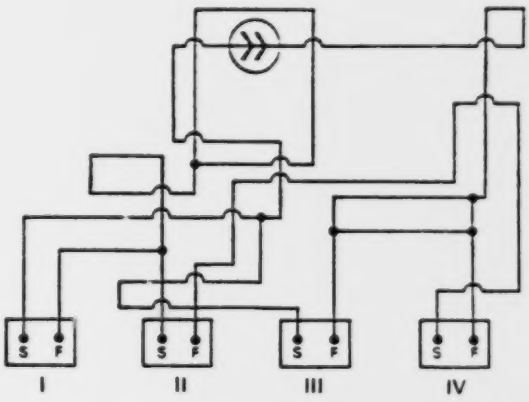
A

C

B

D

10) When travelling along the paths of this network, changes of tracks are possible only at intersections marked with a dot (•).



From which one of these boxes can you start at S, follow some path which passes through the circle and return to F in the same box?

A box I

B box II

C box III

D box IV

11) The shape to the left of the dotted line can be made by fitting together four of the five shapes without overlap. Which one is **not** needed?

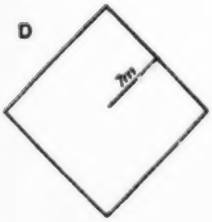
12) In each of these squares, one of the dimensions has been marked to represent 7 metres. Which one of the squares would have the greatest perimeter?

A

C

B

D



13) If m stands for any number, 4 × m can be written as

A m + m + m + 4.
B m ÷ 4.
C m ÷ m + m + m.
D m + 4.

14) A school has one teacher for every 22 pupils. If the school has 88 teachers, which one of the following is the best estimate of the number of pupils attending the school?

A 352
B 700
C 1500
D 1900

15) Jimmy has eaten one piece of pie, as shown in the diagram.

He decides to cut the rest of the pie into pieces the same size as the piece he ate. How many pieces of pie will there be now?

A 4
B 5
C 6
D 7
E 9

16) Which one of the following diagrams does not have $\frac{1}{3}$ shaded?

A

B

C

D

17) In a magic square, the entries in any row, column, or diagonal always add up to the same number. Some entries in this 3 × 3 magic square are missing.

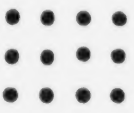
5	19	x
	11	
		17

Which one of the following statements is true about x?

A x must stand for 7
B x must stand for 9
C x must stand for 24
D x must stand for 33
E There is insufficient information to find the value of x.

18) 'Rectangular' patterns can be formed from different numbers of dots so that each pattern is at least 2 dots wide.
For example, 12 dots form these two patterns.

pattern 1



pattern 2

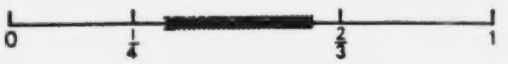
Which one of the following numbers of dots will not make a rectangular pattern?

A 15 C 29
B 21 D 39

19) If half an apple is shared equally by three people, what fraction of a whole apple does each person receive?

A $\frac{1}{3}$ C $\frac{1}{6}$
B $\frac{1}{4}$ D $\frac{1}{8}$

20) The diagram shows part of a number line.



Which one of the following represents a point in the shaded part of this number line?

A $\frac{1}{2}$ C $\frac{4}{5}$
B $\frac{3}{4}$ D $\frac{1}{8}$

21) If □ stands for any odd number, then $(2 \times \square) - 1$

A must be odd.
B must be even.
C. can be odd or even.

22) A girl sent one card each to 21 friends. Each card cost 25 cents, and the postage for each card was 18 cents.
Which one of the following gives the total cost?

A (21 + 25) + 18 cents
B 21 × (25 + 18) cents
C 21 + (25 × 18) cents
D (21 + 25) × (21 + 18) cents

23) For every 3 bottles collected, a girl receives 7 cents. If she collected 210 bottles, which expression would give the total amount of money she received?

A $\frac{210 \times 3}{7}$ cents C $\frac{210 \times 7}{3}$ cents
B $\frac{3 \times 7}{210}$ cents D $\frac{210}{3 \times 7}$ cents

24) The temperature at 8 a.m. was 7.8 °C and by 2 p.m. it had risen by 16.6 C degrees.
Which one of the following was the temperature at 2 p.m.?

A 8.8 °C C 23.14 °C
B 16.6 °C D 24.4 °C

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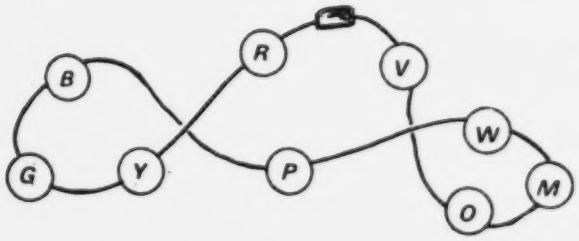

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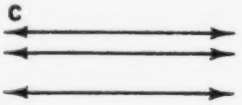
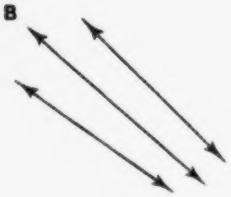
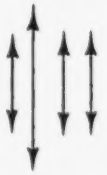
1) Which one of the following is a group of lines which are **not** parallel?

A

C 

B 

D 

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C

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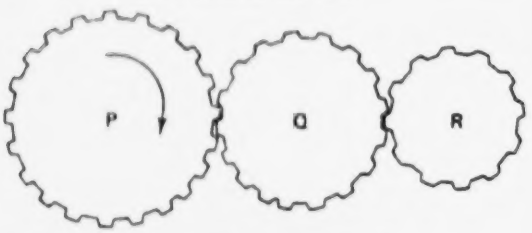

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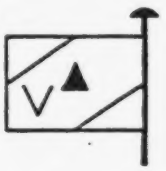
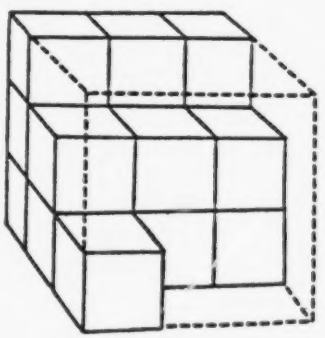
C

B 

D

5) How many more small cubes would be needed to complete the larger cube, as outlined by the dotted lines?



A 8
B 9
C 11
D 14

6) Which one of the following pieces will fit into the blank space and complete the design?

A

C

B

D

7) How many triangles are there in this figure?

A 3 C 5

B 4 D 6

8) Some words, like MUM, look the same if we read them in a mirror placed vertically beside them.

Which one of the following would represent the same number if read in a mirror placed beside it?

A 30103 C 8181

B 696 D 8008

9) This clown's head was drawn on the outside of a balloon.

Which one of the following shows how the head would appear if it could be viewed from inside the balloon?

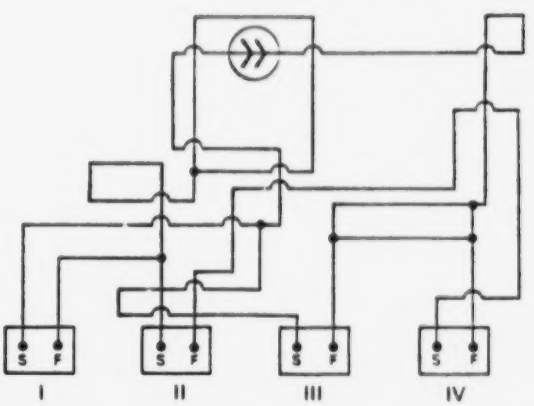
A

C

B

D

10) When travelling along the paths of this network, changes of tracks are possible only at intersections marked with a dot (•).



From which one of these boxes can you start at S, follow some path which passes through the circle and return to F in the same box?

A box I C box III

B box II D box IV

11) The shape to the left of the dotted line can be made by fitting together four of the five shapes without overlap. Which one is not needed?

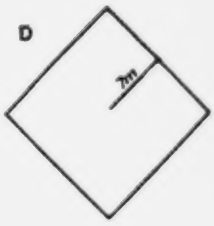
12) In each of these squares, one of the dimensions has been marked to represent 7 metres. Which one of the squares would have the greatest perimeter?

A

C

B

D



13) If m stands for any number, 4 × m can be written as

A m + m + m + 4.
B m ÷ 4.
C m ÷ m + m + m.
D m + 4.

14) A school has one teacher for every 22 pupils. If the school has 88 teachers, which one of the following is the best estimate of the number of pupils attending the school?

A 352
B 700
C 1500
D 1900

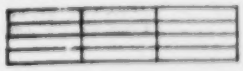
15) Jimmy has eaten one piece of pie, as shown in the diagram.

He decides to cut the rest of the pie into pieces the same size as the piece he ate. How many pieces of pie will there be now?

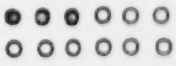
A 4
B 5
C 6
D 7
E 9

16) Which one of the following diagrams does not have $\frac{1}{3}$ shaded?

A



B



C

D

17) In a magic square, the entries in any row, column, or diagonal always add up to the same number. Some entries in this 3 × 3 magic square are missing.

5	19	x
	11	
		17

Which one of the following statements is true about x?

A x must stand for 7
B x must stand for 9
C x must stand for 24
D x must stand for 33
E There is insufficient information to find the value of x.

18) 'Rectangular' patterns can be formed from different numbers of dots so that each pattern is at least 2 dots wide.
For example, 12 dots form these two patterns.

pattern 1

pattern 2

Which one of the following numbers of dots will not make a rectangular pattern?

A 15 C 29
B 21 D 39

19) If half an apple is shared equally by three people, what fraction of a whole apple does each person receive?

A $\frac{1}{3}$ C $\frac{1}{6}$
B $\frac{1}{4}$ D $\frac{1}{8}$

20) The diagram shows part of a number line.

Which one of the following represents a point in the shaded part of this number line?

A $\frac{1}{2}$ C $\frac{4}{5}$
B $\frac{3}{4}$ D $\frac{1}{8}$

21) If $\square$ stands for any odd number, then $(2 \times \square) - 1$

A must be odd.
B must be even.
C can be odd or even.

22) A girl sent one card each to 21 friends. Each card cost 25 cents, and the postage for each card was 18 cents.
Which one of the following gives the total cost?

A (21 + 25) + 18 cents
B 21 × (25 + 18) cents
C 21 + (25 × 18) cents
D (21 + 25) × (21 + 18) cents

23) For every 3 bottles collected, a girl receives 7 cents. If she collected 210 bottles, which expression would give the total amount of money she received?

A $\frac{210 \times 3}{7}$ cents C $\frac{210 \times 7}{3}$ cents
B $\frac{3 \times 7}{210}$ cents D $\frac{210}{3 \times 7}$ cents

24) The temperature at 8 a.m. was 7.8 °C and by 2 p.m. it had risen by 16.6 C degrees.
Which one of the following was the temperature at 2 p.m.?

A 8.8 °C C 23.14 °C
B 16.6 °C D 24.4 °C